Optimal algorithm for phase shift searching for the DPSBF

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***Abstract*—The distributed phase shift beamforming is the great technique for spatial signal filtering in wireless networks. We already showed that this technique can be exploited for the interference reduction in wireless networks. This paper deals with optimal phase shift search for this method.**

I. Introduction

The distributed phase shift beamforming method needs to be supported by the hardware and software working closely together. This paper optimizes the software part. The one of the main problems is the proper phase shift search. It can be done by brute force however this attitude is almost always time consumptive. If the distributed phase shift beamforming technique is used in the connected network, they can be exploited the existing connections for optimization of phase shift search. This paper propose the modified binary search algorithm which should significantly decrease the complexity of the phase shift searching.

In the world of wireless networks are many of techniques to create some type of beamforming. The basic principle of our type of beamforming is described in the second chapter of this paper. The third chapter propose the new optimal phase shift searching algorithm whose detailed analysis is in the following fourth chapter.

II. Problem definition

A. Network Model

Let us first consider the pure mathematical network model: the network topology can be considered as an undirected graph $G = V \times E$ where the set of vertices V represents the set of hosts and the set of edges E represents the connections between them.

Hosts in V are randomly distributed over a flat surface (thus we take into account only the horizontal plane of the antenna radiation patterns) and all of them are identical. Because in this paper we compare the improvement brought by interference cancellation towards interference in the same network without cancellation (in the same conditions of propagation and with identical transmitters) we can simplify the model by supposing that there are no obstacles involved and no ground reflection therefore the signal propagation is ideal. Every host has only one data transceiver connected to one ideal antenna (gain is 0 dBi in all directions).

The energy source in this model is great enough to be considered as unlimited thus we do not apply models for energy conservation, being oriented towards interference only.

Edges in E represent connections between hosts. Any edge between two hosts can be established only if each is in the transmission range of the other.

B. Distributed Phase Shift Beamforming

In recent years many varied techniques which use different types of beamforming were developed. The other name for such techniques is even more precise: spatial filtering. With beamforming it is now possible to precisely determine the spatial destination of the transmission. Beamforming can be established by using sector turning antennas or by using multiple sector antennas which can be switched on and off on a per packet basis. The same result can be obtained by using antenna arrays [2]. Our work is derived from the antenna array results.

The antenna array is a set of antennas (in many cases omnidirectional) which are connected to a shared transceiver and they transmit the same signal simultaneously. The positions of all antennas are known and, in most cases, the distances between them are comparable to the wavelength which is used for the transmission. The resulting radiation pattern is affected by the multiple signals addition. There are directions where the signal is amplified (constructive interference) and directions where is attenuated (destructive interference). The resulting amplitude change thus depends on the mutual phase difference.

C. Data transmission

The effect of the antenna array can be achieved in a distributed way by independent modules equipped with only one data transceiver and one antenna. In this case there is no central transmitter and the positions of all antennas are unknown. This distributed antenna array needs to be synchronized by using a second transceiver (not used for data transmissions) in order to transmit synchronously with certain signal phase combination. Explanation of the synchronising mechanism is beyond the scope of this paper and it is described in [3] and in the granted patent [4]. The mutual phase shift between transmitters affects the resulting shape of the radiation pattern. In figures 1 and 2 two radiation patterns which were created by two transmitters can be seen. The distance between the transmitters is 30λ and the only difference between them is the mutual phase shift.

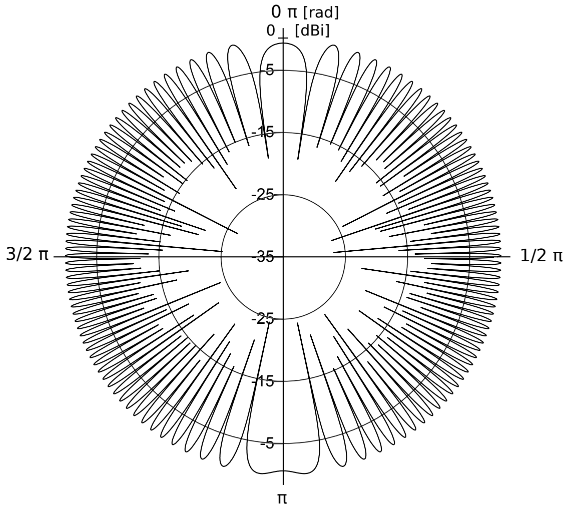

Fig. 1. Radiation pattern of two transmitters with phase shift $\pi/3$ and distance 30 wavelengths

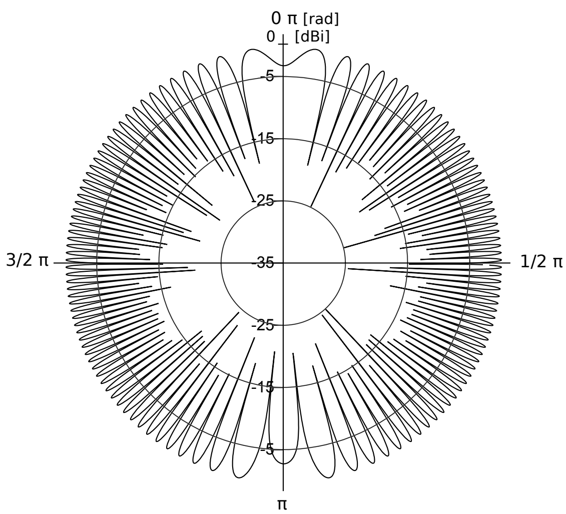

Fig. 2. Radiation pattern of two transmitters with phase shift $\pi + \pi/3$ and distance 30 wavelengths

In this work we will use only two transmitters to create the transmission using DPSBF. The proper timing of both synchronized transmitters is crucial for the successful signal delivery to the destination. The hardware synchronizing mechanism for DPSBF is described in [3], [4] and its explanation is beyond the scope of this paper. In this text we simply assume that synchronization is fully reliable. The transmission using DPSBF takes the same time as the classic one and it can be received be the classic receiver without any special hardware. There is only one condition - the data for the transmission has to be present on both transmitters.

D. Interference Cancellation

As it can be seen in the figures 1 and 2 the resulting radiation patterns using DPSBF do not cover the same area as the single transmitter (whose coverage area is modelled by a circle due to the ideal antenna). This spatial filtering allows the transmission to reach the destination host and, in the same time, can cover a smaller area than the single transmitter.

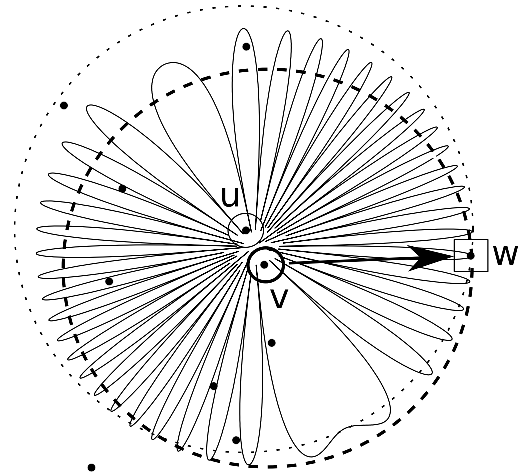

Fig. 3. Example of transmission without DPSBF and with DPSBF

Let us consider the situation in the figure 3 which contains one zone of a wireless network having several hosts. Two of them are denoted by circles (the transmitters u and v) and they are together transmitting by using DPSBF to the destination host (receiver w) which is denoted by the a square. If the DBSBF would not have been used, v would have been used for the classic direct transmission. The continuous line marks the range of both transmitters transmitting together by using DPSBF and the dashed circles mark the range of the individual transmitters without cooperation (u and v). In all cases the transmitting power is set to the minimal value which is sufficient for reaching the destination receiver.

From the figure is obvious that the transmission using DPSBF covers completely different surface and in this case is more convenient than the classic transmission because less other hosts are affected by the transmission. More information about this topic is in [1].

Figure 3 shows us that in certain areas in the network topology it is more convenient to use transmission with DPSBF instead of the classic one. The stronger dashed circle denotes the transmission range of v. We can see that six hosts are covered by this transmission. If we use both transmitters together in DPSBF, the resulting radiation pattern covers only four of other hosts. It means that two more hosts are able to receive other transmission in the same time (spatial filtering).

In figure 3 we can see that the transmission with DPSBF covers the uppermost hosts which were not covered by the classic transmission. In this case the overall result is better (lower interference) however there could be cases in which using DPSBF leads to worse results (higher interference). The solution to this problem is to use classic transmission or DPSBF depending on the results achieved by each method: as TC replaces long edges in the graph with shorter ones only when the shorter ones are better (the metric depends from TC algorithm to algorithm) we can replace edges by DPSBF transmission only when the interference of DPSBF is lower than the original edge caused interference.

By this technique is possible to improve the quality of some edges from the interference view. Not all edges can be improved because the DPSBF is not applicable in all situation.

Unfortunately we do not know in advance which edge can be selected for the DPSBF. Therefore it is necessary to check every edge whether it could be improved.

E. Phase Shift Searching

In order to be the DPSBF applicable, it needs to be properly configured however the finding of the proper phase shift is a time consumptive process. Unfortunately we need to find the best phase shift as possible. It is due to minimizing of the resulting radiation pattern surface. With this pattern is possible to reach the destination with minimal power and in the same time to cover minimal area by the signal. Let us consider that there can be active only one searching process in whole network. (It is possible to execute more phase searching in the same time in one network however one execution of the searching can affect another one if they are in the transmitting range. The solution of this problem is a topic for another paper.) According to this assumption is necessary to execute every searching individually for every edge in the network. Furthermore the transmitting hosts can have multiple neighbours and every of them can be used for DPSBF and every needs individual phase shift searching. Every host x is the neighbour of host u when:

$$\{\forall x \in V \setminus \{u\} : (ux) \in E\} \quad (1)$$

where (ux) means the edge between u and v. If exists the edge between hosts in E, they are neighbours. The searching can be done by brute force. At first is defined the minimal shift step (MS). After it can start searching. For every possible phase are sent data to destination (for example to the host w in the figure 3) and the number of this transmissions is $360/MS$. The host w collects the received signal quality during whole searching and at the end of searching is asked by the host v to send back the best result. After it the host v knows best phase shift between itself and the neighbour host which was used for the searching. The number of all transmissions over the whole network during network initiation can be roughly computed:

$$trNumber = 2 * |E| * (avgNeigh - 1) * (360/MS + 2) \quad (2)$$

where $trNumber$ means overall number of all transmissions to be executed, $avgNeigh$ is the average count of neighbours for all hosts in network. It can be computed: $avgNeigh = 2 * |E|/|V|$. Explanation of all three members of this equation:

1) **2 * |E|** - the doubled number of all edges in the network. The searching needs to be done for all edges in the network however the results are different for both sides of the edge.
2) **(avgNeigh - 1)** - every couple of hosts gives different results. The phase searching needs to be executed for all neighbours of the host which initiates the transmission with DPSBF.
3) **(360/MS + 2)** - number of all possible phase shifts. The two additional transmissions are request for the best result to the destination and answer for this request.

To minimize the count of all transmission during the network initiation there is very few possibilities. Lets discuss about all three parts:

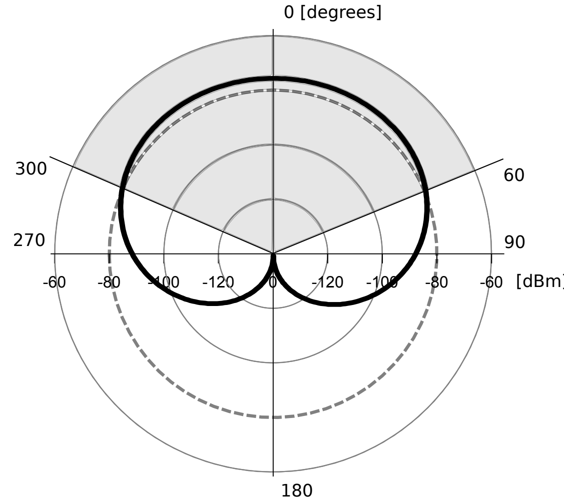

Fig. 4. The dependency of the received power [dBm] on the phase shift [degrees] with using two host for transmission with DPSBF

1) The number of all edges can be decreased for example by some topology control algorithm. This solution unfortunately changes the network topology and it can not be always possible.
2) This part could be improved by a smart neighbour selection. The neighbours which are closer to the original host are in most cases more convenient for the transmission with DPSBF. The smart selection of neighbours has no great effect to the overall complexity especially in the denser networks. Furthermore if we remove some neighbours from the searching, the resulting network performance could be negatively affected.
3) The value MS is crucial for the resulting number of transmissions for this part of equation. The increasing of MS decreases the transmission number however in the same time it is significantly decreased the chance for finding the good phase shifts. In fact we want to have the MS smallest as possible.

From the discussion is obvious that most interesting is the third member of the equation. The MS value affects rapidly the quality of result and overall number of transmissions. The MS value is inversely proportional to the overall number of transmissions.

III. Proposed Solution

In the previous chapter we showed that searching for convenient phase shift for all edges is a complex problem. In the figure 4 can be seen the dependency of the received power on the phase shift used for transmission with DPSBF. The chart is made from the view of the receiving host (for example the w in the figure II-D). The dashed circle denotes the sensitivity of the receiver and the signal power above this line can be received. This sector is grayed. In the situation in the figure 4 is the maximal power for the phase shift 0 degrees. In order to minimize the number of trials by the searching process, we can add immediate feedback from the host w to host u. The host u is able to change the phase shift according the information from the host w. It knows immediately if the last trial transmission improved or decreased the signal power on the host w. With this knowledge it can tune the phase shift towards the maximum received power. In the next few chapters is described the algorithm based on the previous facts. This algorithm is composed of two stages.

A. Stage 1

At the beginning the both transmitting hosts set the power to the maximum. In the Stage 1 the algorithm needs to find such phase shift for which is signal received on the destination host. It does not matter if the signal is strong or weak. Most important is that the signal is received. Look at the figure 4. We only need to find phase shift from the area where is the signal above threshold, so it could be any value from 300 degrees to 60 degrees.

Lets define the searching step (SS). This step begins at maximum possible value - 360 degrees and it is decreased to the half at every iteration of the algorithm. The Stage 1 logic can be seen in algorithm 1. There is defined the set $used$ which should contain all phase shift values which were tried by the algorithm. The main loop continues as long as the signal is not received on the destination host. The Stage 1 finishes when the signal is received on the destination host or if the SS is less than MS. If the signal was not received until the $SS > MS$, the desired phase shift was not found. This situation can possibly occur when the MS is too great and the desired area is too small.

The Stage 1 has two loops. The outer loop ensures termination of whole algorithm. The inner loop sends sets of messages to the destination in such way that phase shifts uniformly cover whole range of 360 degrees and their density decreases at every iteration. The function *send()* has two parameters. First is the phase shift and second is the identification of the destination host. This function distributes the data to the second transmitter and executes the transmission using DPSBF. Into data, which are sent to the destination, is encapsulated information about current phase shift so the destination host can identify the best incoming signal and it can send this information back to the source of transmission. Every message sent by DPSBF contains the information about current phase shift. The destination host keeps this information until is asked to send it as the answer to the source host.

B. Stage 2

In the Stage 1 we searched the phase shift for which is signal received on the destination host. The Stage 2 is looking for the best phase shift which gives a greatest signal power on the destination. Look at the figure 4. The Stage 1 found the phase shift which is somewhere in the grayed area. In this stage we want move the phase towards 0 degrees because there is in our example maximum of power.

The Stage 2 keeps the SS value from the previous stage and sets the current phase to the best value which was found during the first stage. The SS is decreased by half at every iteration and this step is used for exploring the smaller and smaller neighbourhood of the current phase shift. In every iteration are sent two trial transmissions and after that is destination host asked to send the result. In the answer is information whether were the new phase shift better or worse. See the algorithm 2.

Algorithm 1 Stage 1

$SS \leftarrow 360$ ▷ Current searching step size
$used \leftarrow$ empty set ▷ Set of all checked values for SS
while $SS > MS$ **do**
    $p \leftarrow 0$
    for $p < (360/SS)$ **do**
        if $s \notin used$ **then**
            send(s, destination)
            $used \leftarrow s$
        end if
        $p \leftarrow p + 1$
    end for
    $SS \leftarrow SS/2$
    ask the destination host for the results
    if destination received signal **then**
        $phase \leftarrow$ best received phase
        break
    end if
end while

Algorithm 2 Stage 2

$phase \leftarrow$ best confirmed phase from the Stage 1
SS remains unchanged from the Stage 1
while $SS > MS$ **do**
    send($phase + SS, destination$)
    send($phase - SS, destination$)
    ask the destination for the result
    according the confirmation do:
    $phase \leftarrow [phase + SS|phase - SS|SS]$
    $SS \leftarrow SS/2$
end while

C. Optimal Algorithm for the Searching of Phase Shift for the DPSBF

The final algorithm is very simple and it is composed of both stages. See the algorithm 3.

Algorithm 3 Composition of Both Stages

$phase \leftarrow 0$
execute the Stage 1
if the Stage 1 was successful **then**
    execute the Stage 2
    return $phase$
else
    ▷ The Stage 1 did not find the usable phase shift
    return ERROR
end if

IV. Algorithm analysis

The main purpose of the proposed algorithm is decreasing of communication and time complexity. In this chapter we will analyse both stages of the algorithm. In the following expressions the n equals to $360/MS$ and it is the number of transmissions which are sent by the DPSBF. Before we analyse our algorithm, we should know the complexity of the brute force solution. If we use it, the maximal possible number of transmissions is executed so the complexity is:

$$O(n) \tag{3}$$

A. Stage 1 Complexity

At first lets begin with best case. The main purpose of the Stage 1 is to find some phase shift which allows the signal delivery to the destination. It is possible that the first trial transmission is successful. In the first iteration is sent only one message to the destination. After every iteration is sent request for the results and the answer. It gives together three transmissions. In the best case is the complexity:

$$\Omega(1) \tag{4}$$

The worst case is the situation when the success comes in the last iteration (or if there is no success). In the first iteration is sent one message (the same like in the best case). In the second iteration is sent one message too. Every next iteration is sent the doubled number of messages of the previous iteration. After every iteration are sent two messages - request and answer. We can express the overall number of all transmission as:

$$3 + \sum_{i=0}^{\log_2(n)} (2^i + 2) \tag{5}$$

The complexity in the worst case is:

$$O(n + log_2(n)) \tag{6}$$

In the worst case needs to be executed n transmissions using DPSBF and $2log_2(n)$ classic transmissions for request and answer messages after every iteration. After comparison with complexity of the brute force solution (equation 3), we can see that the complexity of the Stage 1 is worse. In the following sections we will show that worst case never occures.

B. Stage 2 Complexity

The analysis of the Stage 2 is more easier. In the best case are executed four transmissions (only one iteration is executed). The number of iteration is in the worst case $\log_2(n)$ and every iteration are executed exactly four transmissions. The complexity equals to:

$$\Theta(log_2(n)) \tag{7}$$

C. Algorithm Complexity

In the figure 5 we can see the first four iterations of the Stage 1 part of the algorithm. The finite number of algorithm steps depends on the size of the mutual phase shift between both compounding signals.

Theorem 1. *Stage 1 of the proposed algorithm finishes the most in two steps in all possible cases.*

Proof: As we can see in the figure 5 that the first part of the algorithm finishes in two steps only if the range of the phase shift between signal is equal to or bigger than 180 degrees. Let us set one of the signal as the reference. If we add any other signal to the reference and the mutual phase shift between signals will be between 0 and 90 degrees, the resulting signal will be always greater than the original reference signal (see the figure 6b, 6c). So now we can see that the we always have 180 degrees range for the successful signal transmission to the destination therefore the we can do that only in two trials. ■

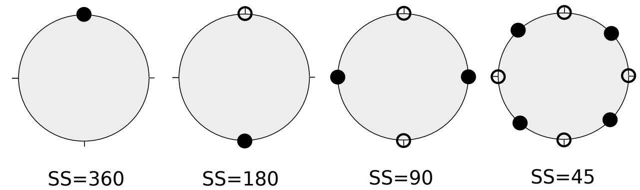

Fig. 5. Execution of Stage 1

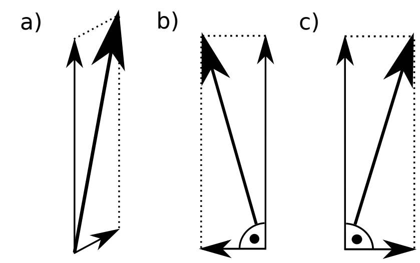

Fig. 6. The signal addition

Now we can put together complexity of both stages. According the previous complexity equations and the theorem, we know that Stage 1 complexity is $O(1)$ so the overall complexity equals to the complexity of the Stage 2:

$$\Theta(log_2(n)) \tag{8}$$

V. Conclusion

We showed that the complexity searching of the phase shift for the method DPSBF can be decreased from the linear to the logarithmic complexity. Using modified binary search algorithm can really speed up the initial process of the network using interference cancelling by usage of DPSBF. In case of periodic network reconfiguration could the proposed method conserve the energy of hosts.

Acknowledgment

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